Optimization of an accelerated step-stress fatigue test plan

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Abstract The design of a component subjected to fatigue requires an experimental qualification of the couple material/process fatigue characteristics. The current industrial requirements involve that this qualification has to be carried out on the smallest possible number of tests and within the smallest possible testing time to censorship. In the same time, the accuracy of estimations has to remain significant.
To do this, accelerated step-stress fatigue tests are quite efficient. Moreover, the accuracy of this kind of test can be optimized thanks to a good distribution of the steps.
We suggest illustrating on a particular example the process to obtain an optimized test plan by this way. Its efficiency will be compared to that obtained with other kinds of plans and the advantage of this method will be shown out for the design of a component. To begin with, the existing accelerated fatigue testing methods and their application to fatigue design, such as the statistical process of results will be reminded. The use of the strength distribution obtained to estimate the reliability will be sum up. Then, the advantage of a step-stress fatigue test with a common profile applied to any tested specimen will be detailed. The optimization process used to define this profile will be explained. It will be shown how its efficiency can be evaluated by stochastic simulations. A real test will be defined with this optimized profile and the specific parameters will be estimated (Basquin's parameters, standard deviation, …).
The efficiency obtained with this test will be compared to that obtained with existing testing method. The focus will be put on the testing times and the sample sizes required with this method and with the other ones to obtain a significant accuracy level. To show out the applicability of the method and to evaluate its efficiency, the method will be carried out to design a actual mechanical component (a window mechanism) with a reliability approach.

1 INTRODUCTION

The evolution of the industrial competition requires designing mechanical components faster and faster. This requirement is particularly difficult to fulfil when the fatigue strength has to be qualified because, the stress levels subjected to the component are often quite low. For this reason, there is hardly any evolution of the failure probability during the first thousands of cycles. Thus, the early stage of the test is useless.

Moreover, to take into account the stochastic behaviour of the material, probabilized SN curves have to be drawn, which requires information about the standard deviations. The problem is that an accurate estimation of a standard deviation often requires a lot of specimens.

All these difficulties lead to extensive costs and testing times. To face these problems, many works **[1] [2]** have suggested using Accelerated Life Testing (ALT). To prevent the useless testing times in the beginning of the test, the key is to increase the stress level. The first kind of ALT consists of testing the specimens at several high stress levels and to extrapolate the fatigue lives obtained to lower levels (**Figure 1**).

More recently, Elsayed has shown that step-stress ALTs are more efficient **[3]**. This second kind of ALT consists of (**Figure 2**) testing all the specimens at a high stress level during a given number of cycles. Then, the survival specimens are tested at another level and so on. A number of cycles to censorship may be defined to limit the testing time.

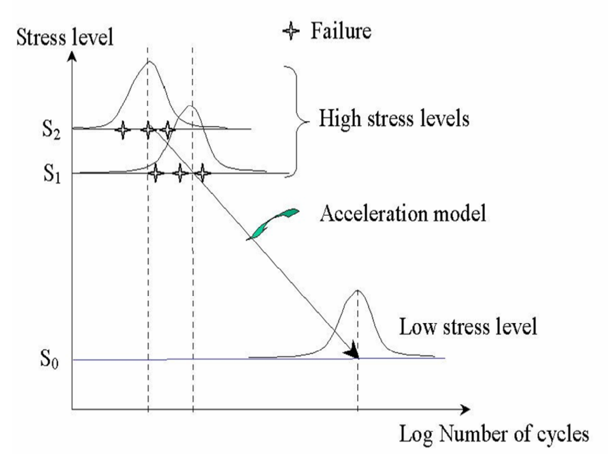

Figure 1. Principle of the usual ALT

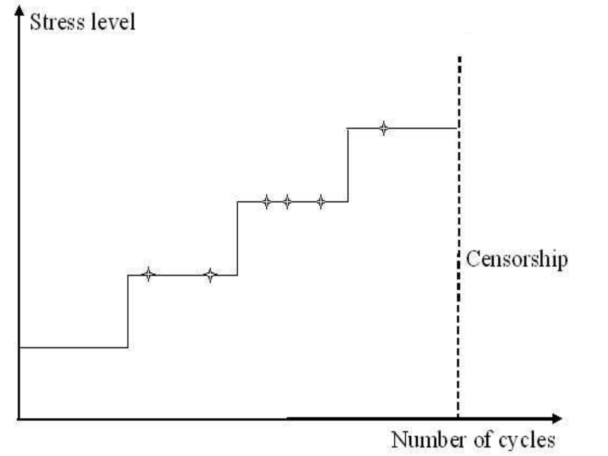

Figure 2: Step-stress fatigue test

Of course, these tests require preventing any modification of the damaging mode (from the low level to the higher levels chosen).

Moreover, an acceleration model has to be known, that is a parametric relationship between the mean or median fatigue life at a given stress level and the value of this stress level. For example, the fatigue model usually defined for high fatigue of metals is an inverse power model (Basquin model). It means that:

$$N(S) = \frac{a}{S^b} \tag{1}$$

where:
S is the constant amplitude stress level,
$N(S)$ is the median number of cycles to failure at the constant stress level S,
a and b are specific parameters usually called Basquin parameters.

The class of statistical distribution of fatigue lives has also to be known. For example, the fatigue life at a given stress level S in the high fatigue domain is usually supposed to follow a lognormal distribution **[5]**. It means that the probability of non-failure at a given fatigue life n is given by:

$$P_{NR}(n) = 1 - \Phi\left(\frac{\log(n) - \log(N(S))}{\sigma}\right) \tag{2}$$

where:
$N(S)$ is the median fatigue life at the stress level S,
Φ is the cumulated distribution function of the standard normal distribution,
σ is the standard deviation of $\log(n)$ at the stress level S.

In the high cycle fatigue domain, σ can be supposed constant for any stress level S above the fatigue limit.

For the first kind of ALTs, tests' results at high levels are fitted to estimate the parameters a, b and σ and then the previous relationships are used to define the fatigue life distribution at the lower stress level.

As far as step-stress ALT are concerned, the fatigue life prediction often uses damaging models such as the linear Miner model **[6]** but experiments have shown that it was very inaccurate and that more complicated damaging models were quite hard to use in practice **[7]**.

For step-stress ALTs, the Sedyakin's model **[8]** already used by Nikulin and Bagdonavicius **[4]** to process the results of a step-stress ALT seems to require fewer assumptions. It consists of defining the evolution of the probability of non-failure for the strep-stress test from the evolution of this probability for each single stress level, as shown on **Figure 3**.

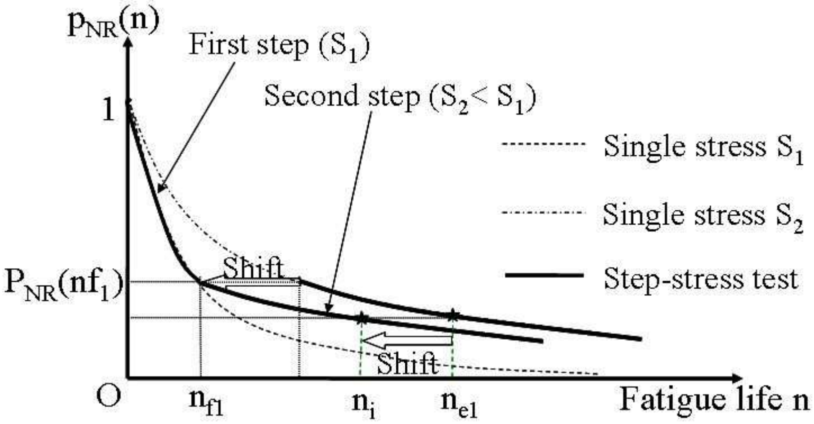

Figure 3. Illustration of the Sedyakin's model

This method lies on two assumptions:

1- The evolution of this probability at each step-stress changing is continuous.

2- From this probability's value at the beginning of a given step, its evolution only depends on the stress level at this step.

At the end of any ALT, Least Square **[9]** or Maximum Likelihood **[4]** estimates of the parameters can be obtained. The accuracy of estimates can also be estimated by the use of bootstrap methods or the covariance Fisher information matrix.

With this method, it has recently been suggested to optimize the distribution of the steps' levels and durations **[9] [10]**.

One has to evaluate by simulation a lot of test plans and to select the best one, which can be done by random generation of the fatigue lives. The problem is that the number of simulations required may be very high. Thus, it may be useful to concentrate on the most potentially efficient test plans.

We suggest concentrating on plans for which the number of failures observed for the different specimens is equally distributed among the different steps.

To do this, the following process can be carried out:

1- A first estimation of parameters is obtained from short tests (tensile test, fatigue test beneath the low cycle fatigue domain).
2- The evolution of the non-failure probability is defined at one single stress level from these first estimations.
3- The times corresponding to several fractiles of the number of failures are calculated
4- These times are changed into ending step times leading to the same fractiles. The first estimations of parameters enable one to calculate them.
5- The step-stress plan obtained is tested by several simulations to see the influence of the stochastic behaviour and to verify the robustness of this test plan. To do this, the values of parameters estimated after each simulated test at the nominal level chosen are compared to input values.

When the accuracy is ensured, the real step-stress fatigue tests are carried on the available specimens. Then, the fatigue lives obtained are processed by Maximum Likelihood Estimation (MLE) and the accuracy of these estimations is estimated too (by the Fisher information matrix).

This method can be applied to quickly qualify materials from tests on smooth specimens but it can be done on actual mechanical components too. If short tests results, bibliography, Finite Elements simulations, or past tests' results on similar components are available, it enables one to obtain first estimations for the component of interest, and thus to define an optimized test plan.

To validate this method, this plan has been carried out on smooth specimens made of stretched C38 steel and on an end stop window's mechanism. The detail of the experiments and the results obtained are detailed in the following. Its efficiency has been verified by comparison with a classical fatigue test plan.

2 EXPERIMENTS ON SMOOTH SPECIMENS

2.1 Short tests

The purpose of short tests was:

- to identify in the bibliography the relevant informations about the material used,
- to bound the high cycle fatigue domain,
- to have a first estimation of the fatigue limit S_D, the Basquin parameters and the standard deviation σ of log(n), where n denotes the fatigue life at a given stress level in the high cycle fatigue domain.

First of all, one or more tensile tests can indicate the ultimate strength and the yield strength. Thus, the upper limit of the high cycle fatigue can be located.

A hardness test enables one to verify that the limit obtained is consistent with the material hardness.

The microscopic observations indicate the grain size and shape. Thus, the heat and mechanical treatments subjected by the specimens can be known.

At last, some fatigue tests at a stress level S_1 slightly beside the yield stress R_e give an estimation of σ and a mean value μ_1 for log(n) at this stress level.

The relationship (1) represents a linear evolution of log(n) with respect to S. Thus, this evolution can be plotted by a straight line joining the two points (μ_1 , S_1) and (log(10^7),S_D) and then, the estimation of the Basquin parameters a and b is obvious.

At the end of these short tests, approximations of all the parameters required to simulate any step-stress test in the high cycle domain are available and, the AFT model used by Bagdonavicius and Nikulin **[4]** may change any number of cycles n for a step-stress test into an equivalent number of cycles n_{equ} defined for a test at a single stress level (and the inverse transformation is possible too).

Let us remind that, for two different fatigue tests, two testing numbers of cycles are told equivalent if the probabilities of failure at these two numbers of cycles are equal. Thus, the step-stress test can be defined so that the same fractile of failures is defined at all the steps and it can be verified by simulations in which random values corresponding to the step-stress test found are generated (with the estimates of a, b and σ obtained after the short tests). For the C38 steel used, tests on smooth specimens give the following results (**Table I**).

Table I. Estimations from short tests

R_m	R_e	S_D	S_1	μ_1	a	b	σ
810 MPa	605 MPa	305 MPa	545 MPa	log(9.7 10^3)	0,08	1110 MPa	0,08

2.2 Definition and simulations of the step-stress tests

From the estimations of µ1 and σ, we have plotted the probability of non-failure at the stress level S1 and 5 fractiles (on the number of cycles n) n_{fi}, i = 1 to 5 have been defined. These fractiles have been changed into 5 final numbers of cycles n_{fi}, i = 1 to 5 for each step i as shown on Figure 3. With the estimations in **Table I**, it led to the values in **Table II**.

Table II. Fractiles on n and corresponding final numbers of cycles n_{fi} for the step-stress test.

i	1	2	3	4	5
f_i	0,1	0,3	0,5	0,7	0,9
n_{fi}	32 050	33 820	36 240	44 140	50 500

Figure 4 (in appendix) shows how the failures on 20 smooth specimens simulated are distributed. It led to hardly the same distribution on the different steps for 10 simulations, which shows that, despite of the scatter in the behaviour of the material, this profile often leads to a good distribution of the stress levels at the different failures.

Thus, it seems that with a sample size equal to 20 and with a censorship at $n_c = 6.10^4$ cycles, this plan will give the best accuracy. However, it had to be verified by comparison with other plans.

To do this, the software OPTIFIAB designed by Fiabilys can be used. The numbers of cycles n_{fi}, i = 1 to 4 are shifted from -10 %, -5 %, 5 %, 10 % (the fifth one is unchanged because it is the number of cycles to censorship) and the same is done on the stress levels at each step.

Each time, a step-stress test on 20 specimens is simulated 10 times, which leads to a mean estimation of the number of failures at 10^6 cycles and to a mean accuracy. We confirm that the best accuracy is found with the plan defined in table II, which demonstrates that it is an optimal test plan (if the estimations of a, b and σ are not too far from their true values and if the shifts chosen are representative enough).

2.3 Experiment on smooth specimens

A sample of 20 specimens has been subjected to the optimized rotative bending step-stress test shown on **Figure 4**. The distribution of failures obtained is shown on **Figure 5** (in appendix).

Because of the slight difference between the estimated values of *a*, *b* and *σ* and their true values, the fatigue lives observed are not exactly distributed among all the steps but it is hardly what we expected anyway.

To evaluate the efficiency of that plan, maximum likelihood estimates (MLE) have also been obtained from the usual plan on 30 specimens subjected to 5 constant amplitude stress levels **[11]**.

Let us compare the estimations obtained by MLE from these real results to those obtained from simulated results and let us estimate with the information Fisher matrix the coefficient of variation of these estimates (**Table III**).

Table III. Estimations and accuracy level

	a	b (MPa)	σ	S (10^5 cycles)	S (10^6 cycles)
Estimates from short tests	0,08	1110	0,08	442 MPa	368 MPa
Estimates from the usual plan (NF A 03-402)	0,143	2053	0,83	396 MPa	285 MPa
Estimates from the step-stress test	0,14	2059	0,95	411 MPa	298 MPa
Estimated coefficient of variation	4,5 %	3,7 %	17,5 %	5,4 %	7,2 %

Not only the fatigue strength at 10^5 or 10^6 cycles estimated from the optimized plan is close to that estimated from the usual test plan, but also, the estimate of the coefficient of variation enables one to correctly bound the true value (as the usual tests have been carried out on a lot of specimens, we consider the corresponding estimation as a reference).

Moreover, we can see that the important errors on the estimates obtained from short tests don't prevent a good efficiency of the optimized plan. It demonstrates the robustness of the method.

3 – APPLICATION TO MECHANICAL COMPONENTS

The component tested was a guideway used for pivoted windows. The weak point particularly subjected to fatigue was the guideway stop end (**Figure 6**).

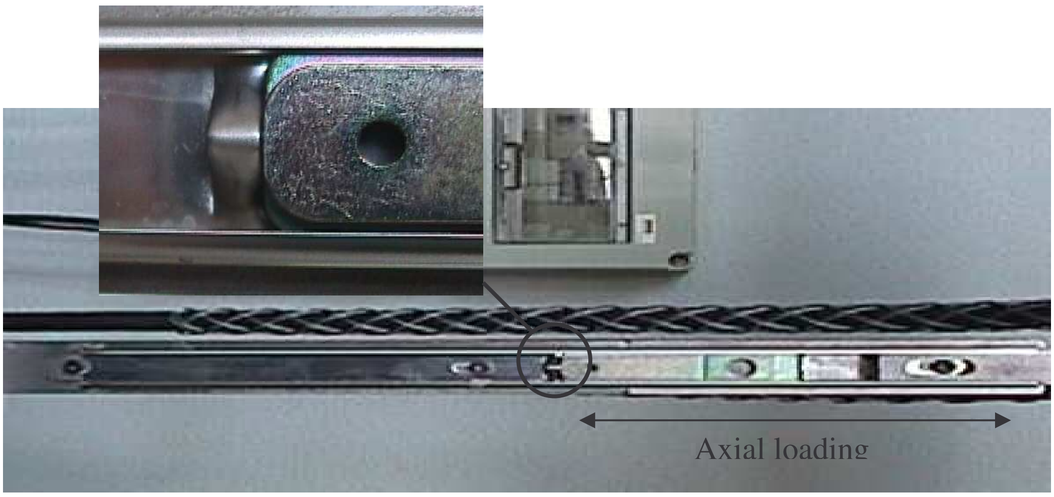

Figure 6. Guideway with its stop end.

In this study, we have defined the optimized test plan from an existing fatigue curve obtained with a usual fatigue test plan at constant amplitude force levels. It had been done by pushing on the guideway stop end with an axial fatigue test machine. These results have shown that the evolution of the pressing force with respect to the corresponding number of cycles to failure had the same shape as the SN curve for steel smooth specimens. The central region could also be modelled by a Basquin relationship, and the standard deviation of log(n) at a given force level can be supposed constant in this region.

Thus, from the parameters obtained, an optimized test plan with successive steps at 5 different force levels could be defined.

This test has been carried out on 5 components only. Thus, the results have not been processed by a simple MLE but we have used a Bayesian approach. It means that an a priori has been used on the parameters to eliminate the unavoidable effect of scatter for such a small sample. In our case, the a priori was a uniform distribution of the three parameters from 80 % to 120 % of the usual estimate obtained on a similar model made of the same kind of steel.

The application of the optimized test plan method can be sum up as follows.

1- From old tests on a similar model of guideway, Basquin parameters have been estimated by

$$\hat{a} = 0{,}08 \text{ and } \hat{b} = 9250 \text{ N},$$

the standard deviation estimate is $\hat{\sigma}=0{,}4$ and the Basquin relationship is correct from F = 2000 N to F = 5000 N.

2- To distribute the failures on all the steps and to ensure the relationship validity, the step-stress test has been defined with the 3 decreasing force amplitude levels:

$$F_1 = 4000 \text{ N}, F_2 = 3200 \text{ N and } F_3 = 2400 \text{ N}.$$

3- The number of cycles at the end of each of these steps has been defined for fractiles of failures equal to 0.3, 0.6 and 0.9. For a test at the constant force amplitude level F_1, It corresponds to the numbers of cycles: 11 000, 17 550 and 24 850. For the step-stress test, it corresponds to numbers of cycle at the end of each step equal to:

n_{f1} = 11 000, n_{f2} = 118 000 and $n_{f3} = 4.5\ 10^6$ (in fact, n_{f3} is replaced by the number of cycles to censorship $n_c = 5\ 10^6$)

Simulations with the software OPTIFIAB have shown that ALT tests with slightly different values of n_{fi} do not lead to a better accuracy for the maximum likelihood estimates. So, the actual tests on the 5 available guideways have been carried out with this test plan.

The results obtained are: 9 630, 67 020, 83 020, 234 520 and $1.382\ 10^6$.

These fatigue lives are well distributed on the different steps. The MLE led to an estimated fatigue limit at 10^7 cycles: F_d = 2430 N with a coefficient of variation equal to 43 % and an estimated standard deviation on log(n) $\hat{\sigma}=0{,}32$ with a coefficient of variation on this estimate equal to 82 %.

The strong inaccuracy on these estimates stems from the very little size of the tested sample. Better estimates can be obtained by the use of the Bayesian a priori defined above. It led to the a posteriori estimates:

$a = 8.43\ 10^{-2}$, b = 9735, F_d = 2509 N and $\hat{\sigma} = 0.384$

As a bayesian a priori was used, the accuracy on these estimates could not be estimated by the Fisher information matrix anymore but it has been estimated from 100 simulations of tests.

To do this, 100 lists of 5 fatigue lives have been defined by random generations with the parameters estimated from the actual tests. Each time, new estimates of these parameters have been obtained with the same Bayesian method and with the same a priori. Because of the random effects of this process, the a posteriori estimates obtained were always different from the input values.

As a comparison, the classical plan of fatigue tests has been carried out with the constant force amplitudes F_1, F_2 and F_3 and with the same sample size and the same number of cycles to censorship. MLE and Bayesian estimates have been obtained (with the a priori defined above) and the coefficient of variation (CoV) has been estimated by the Fisher matrix for the MLE and by 100 simulations of tests for the bayesian estimates. The results are sum up in **Table IV**.

Table IV: Comparison of estimates with the classical plan and with the optimized plan (with the CoV)

	a	b	$F_d(10^7)$	σ
Ref: Usual plan on 100 guideways	0,086 ± 0,8 %	9732 ± 2 %	2503 ± 0.3 %	0.39 ± 8 %
MLE with the classical plan (5 guides)	0,078 ± 11 %	9248 ± 58 %	2107 ± 55 %	0,19 ± 230 %
MLE with the optimized plan (5 guides)	0,079 ± 6 %	9540 ± 39 %	2430 ± 43 %	0,27 ± 82 %
Bayesian estim. (classical plan) - 5 gu.	0,082 ± 5%	9726 ± 5 %	2478 ± 3 %	0,412 ± 21 %
Bayesian estim. (optimized plan) – 5 gu.	0,085 ± 1%	9735 ± 2 %	2509 ± 0.4 %	0.384 ± 11 %

If the reference is the estimation of parameters obtained with 100 components subjected to constant force amplitudes and without any Bayesian approach, we have shown that, for the same bayesian approach and the same sample sizes, the estimates obtained from the optimized plan are more accurate than those obtained with constant force amplitudes.

These acceptable results have required a relevant a priori. For our study, it was possible because a similar model of guideway had been tested before but such results will not be always available. When this is the case, the bayesian a priori may also be obtained from finite elements simulations with the fatigue parameters of the material used, or with short test results or also with values available in bibliography.

4 – CONCLUSION

For a metal fatigue qualification or for the validation of a whole mechanical component subjected to fatigue, the optimization method presented can enable one to save time and money without reducing the accuracy of fatigue lives estimations.

The experiments carried out on steel specimens have demonstrated that the method was not only theoretically justified but also applicable for industrials.

Of course, it would be better validating the interest of the step-stress fatigue tests chosen for other kinds of metals such as aluminium or copper alloys. However, the only caution to take to do this is to verify the accuracy of the Sedyakin and Basquin models. This can be done quite conveniently by preventing any plastic strain during the test (with measures of temperatures for example) and, anyway, it is possible to control at the end of the tests if the final results are consistent with these models.

As far as tests on whole mechanical components are concerned, this method will be particularly useful because, for the moment, it is very hard to obtain accurate estimations of standard deviations with the usual sample size available. It leads to overestimate these standard deviation and thus to overdimension the component. Sometimes, it requires redesigning the component where as it was correct.

As a conclusion, it is worth designing an optimized fatigue test plan fitted to the material or component tested. Despite of the innovating aspect of the method, there is very little chance to misevaluate the final fatigue strength.

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APPENDIX. Comparison of the simulated test and the actual one.

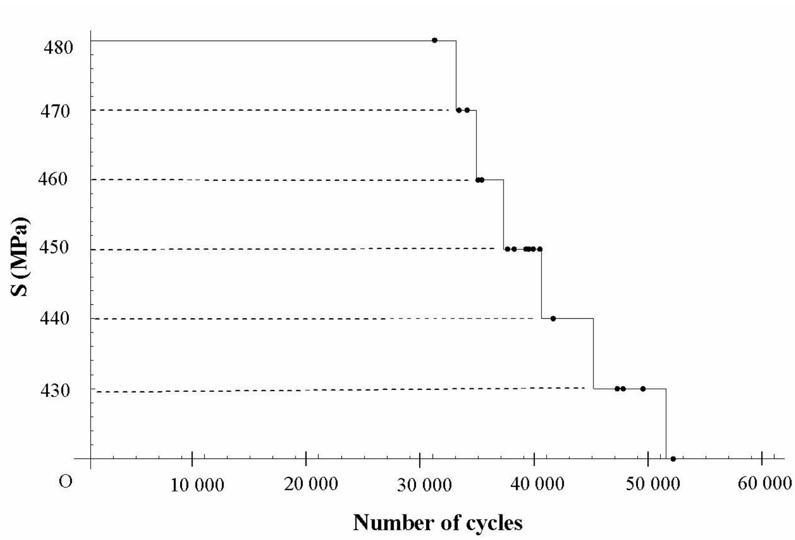

Figure 4. Distribution of the failures for the optimized test plan (15 failures, 5 censored).

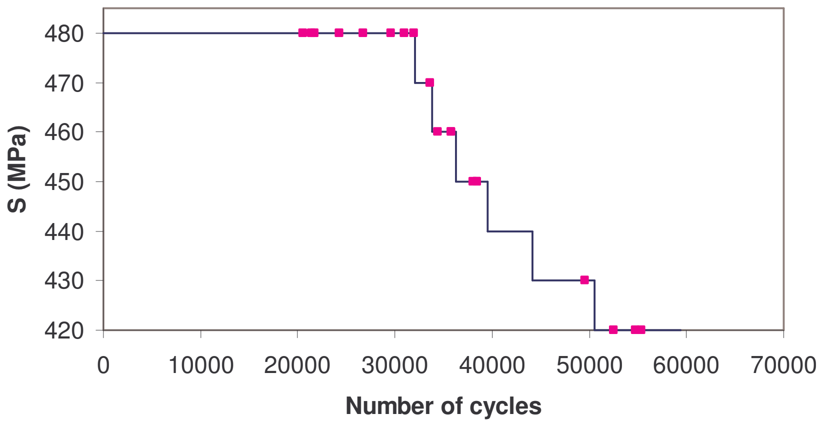

Figure 5. Results of a real optimized step-stress test